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MODELING OF NUTATION OSCILLATIONS IN LIGHT RADIATION REFLECTED BY THIN RESONANT LAYER

E. V. Timoshchenko
Candidate of Physical and Mathematical Sciences,
Associate Professor, Vice-Rector for Research
Mogilev A.A. Kuleshov State University
Yu. V. Yurevich
Candidate of Physical and Mathematical Sciences, Associate Professor
Mogilev State University of Food Technologies

Within the framework of the semiclassical oscillator model the possibility of modulating radiation upon resonant reflection in the regime of coherent interaction of lasing field with a thin-layer substance is predicted basing on methods of the mathematical theory of stability of nonlinear dynamical systems and numerical modeling.

Keywords: thin planar layer of resonant atoms, dipole-dipole interaction, self-sustained light pulsations

Introduction

Collective optical processes capable of developing under conditions of resonant interaction and the relative weakness of relaxation mechanisms are characterized by a high consistency of light field oscillations and polarization of the medium. Factors that can violate the coherence of the field and polarization and significantly complicate the dynamics of this kind of ultrafast optical processes include dipole-dipole interaction. Its arising is typical of a substance with a high concentration of active centers and relatively large dipole moments attractive to these structural elements – the so-called dense resonant media.

It is believed that similar materials are also represented by semiconductor quantum-sized heterostructures that resonantly respond to radiation in the exciton region of the spectrum [1]. Similar structures are also a convenient experimental and theoretical model for studying the dynamics of coherent effects [2; 3]. Based on them, in a thin-film design, nonlinear modulating elements are developed in compact optical information processing devices. The study of the dynamics of their reaction to radiation in a coherent mode of interaction between the optical field and the active medium is therefore a nontrivial and practically important problem.

1. Computation model

In this regard, the problem of modeling the dynamics of reflection of a resonantly polarizable film in the framework of a semiclassical approach using the hyperfine layer approximation was posed [4]. The nonlinear response of the medium is described by the equations of the quantum-mechanical density matrix, and the field (incident from outside E_i, reflected E_r, transmitted E, and acting on dipole atoms) by the relations obtained from the electrodynamic conditions for the fields and the boundary layer with resonant polarization:

$$E_r = -r\,E_i + \frac{\mu N l}{\varepsilon_0(\eta+1)\,c}\frac{d\rho}{dt}, \quad E = \frac{2}{\eta+1}E_i + \frac{\mu N l}{\varepsilon_0(\eta+1)\,c}\frac{d\rho}{dt},$$

$$\frac{d^2\rho}{dt^2} + \frac{2}{T_2}\frac{d\rho}{dt} + \omega_0\left(\omega_0 - \frac{2\mu^2 N}{3\varepsilon_0\hbar}n\right)\rho = \frac{2\mu}{\hbar}\omega_0 n E, \quad \frac{dn}{dt} + \frac{1}{T_1}(n-1) = \frac{2\mu}{\hbar\omega_0}\frac{d\rho}{dt}\left(E + \frac{\mu N}{3\varepsilon_0}\rho\right). \quad (1)$$

Here ρ and n are the variable variables of polarization and population difference, μ is the matrix element of the dipole transition, N is the concentration of active dipoles, T_1 and T_2 are the times of longitudinal and irreversible phase relaxation, η и l are refractive index and layer thickness, r_0 is the reflection coefficient of the layer.

System (1) is modified taking into account the contribution to the field of the near fields of elementary dipoles acting on the active centers. The frequency detuning from the resonance ω_0 then depends on the population difference and, therefore, is non-linear. Model parameters of the medium were chosen for quantum-sized structures based on InGaAs, following the data of [5].

2. Reflection of a quasi-continuous light field

In the case of optical field with continuous envelope probing a resonant film, the reflected radiation field receives an expressed nutational structure. The variant of the quasi-continuous signal of the effect corresponded to the temporal distribution of the bending intensity of the applied field, defined by the dependence

$$e'_i(\tau) = e'_0[\exp(\tau/\Delta\tau) - \exp(-\tau/\Delta\tau)]/[\exp(\tau/\Delta\tau) + \exp(-\tau/\Delta\tau)],$$ (Fig. 1, *a*),

the value of Dτ in this case is determined by the steepness of the growth of tension at the initial stage of exposure.

Figure 1 for a different level of excitation and unsaturated absorption, calculated as $\kappa = \mu^2 N l \omega_0 T_2/\varepsilon_0\hbar c$, shows a time pattern of the intensity of the normalized reflected field $e_r = \mu E_r/\hbar\omega_0$.

Looking for the dependencies, the options for calculating the power of the reflected signal are presented mainly by a envelope series of pulsations, which encompass the high-frequency carrier component and attenuate to an equilibrium value. Power oscillations arise as a result of nutation vibrations of dipole particles representing active centers in the film material matrix. The damping of the pulsations in the interaction scheme with a relatively slow irreversible phase relaxation is caused by a violation of the field coherence and the polarization response of the medium due to the shift in the natural frequencies of the active dipoles due to their mutual influence due to the near fields.

The emergence of the substructure “started” from a certain power value (Fig. 1*, b*; 1, *c*). The increase in the applied power, with other fixed parameters, led to a reduction in the transition period in the output to the ripple mode. At the same time, the frequency of nutation pulsations increased, their contrast decreased (Fig. 4, *b - e).* These regularities of the temporal pattern are to a certain extent similar to how the structure of laser radiation in the free-lasing mode changes with increasing excitation level (pumping rate). An increase in the resonance absorption index, however, changes the picture of the nutational pulsations in another way – the contrast and the duty cycle increases, while the frequency of their repetition decreases (Fig. 1, *g - m*).

The presence of two opposing trends in the development of a picture when these basic characteristics change, which can be changed in the experiment, makes it possible that with a certain combination of them, the optimal variant of the process is possible, when a series of nutation pulsations will be represented by self-oscillations of power in reflected radiation.

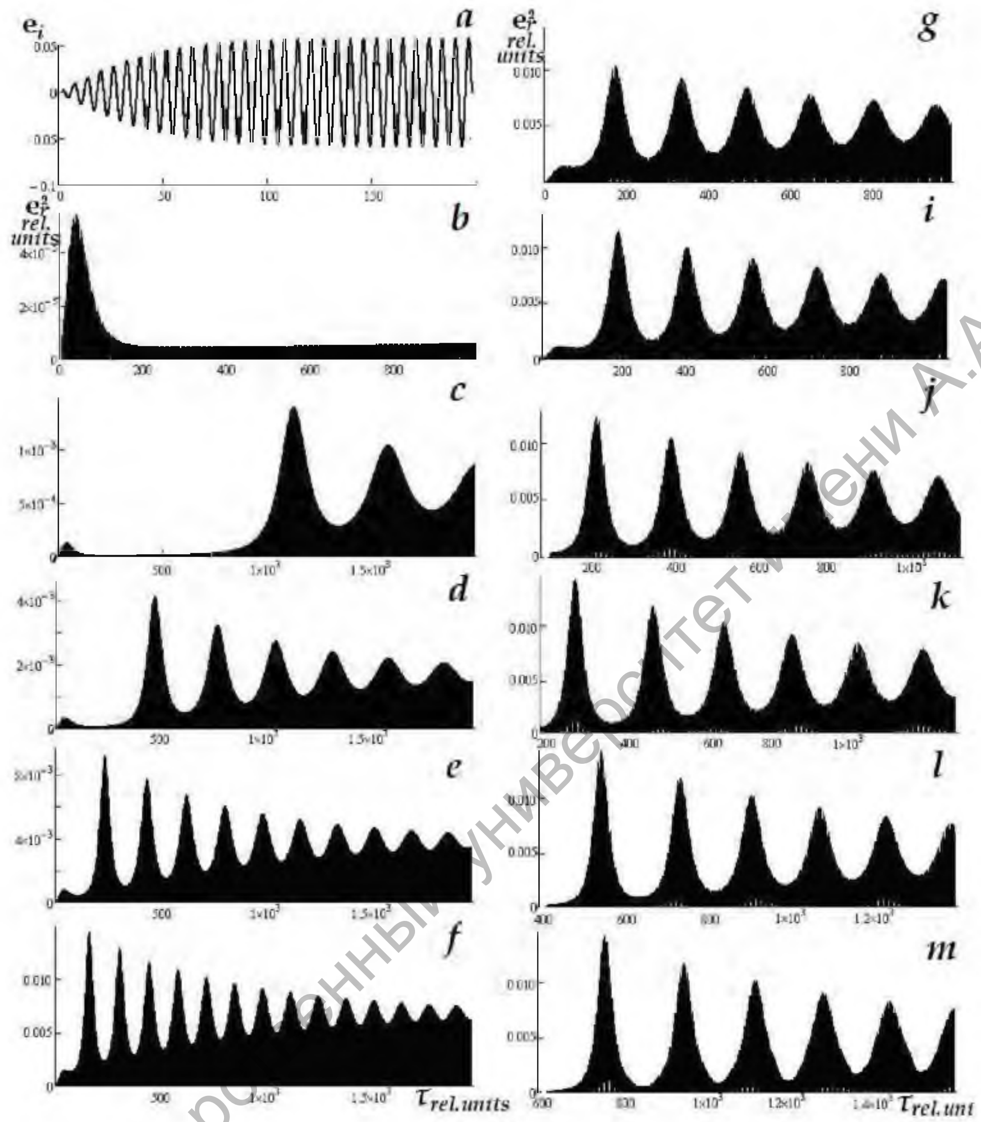

Fig. 1 – Temporal dependence of the normalized intensity of reflected field; *a* is incident signal shape, *b* – e'_0=0.05, *c* – e'_0=0.15, *d* – e'_0=0.25, *e* – e'_0=0.5, *f* – e'_0=1.25, $\boldsymbol{\kappa}$= 2; *g* – $\boldsymbol{\kappa}$= =3.5, *j* – $\boldsymbol{\kappa}$= 4, *k* –$\boldsymbol{\kappa}$= 5, *l* – $\boldsymbol{\kappa}$= 6, *m* – $\boldsymbol{\kappa}$= 9, e'_0= 0.5, γ = 0.0078, $\boldsymbol{\tau}_2$= 400, $\boldsymbol{\omega}_0$= $1.45{\cdot}10^{14}$*rad/s*

For quasistationary envelopes of variables of field strengths and resonant polarization, system (1) is reduced to the optical Bloch equations. Below are the results of the analysis of the equilibrium states of the model in the framework of the mathematical theory of stability.

3. Analysis of the stability of the analogue of the initial oscillator model

The calculated evaluation of the properties of the quasi-equilibrium states of the source model (1) was carried out within the framework of a linear analysis of the stability of the quasistationary analogue of the model. This means considering the dynamical system for relatively slow field envelopes and polarization, that is, similar to [2], the quasistationary approximation of model (1) was used.

The transition to this approximate oscillatory system is trivial; it is formulated for relatively slowly varying amplitudes and fields and polarization. By solving the system with a high degree of coincidence, regularities of the reflection processes calculated for the variants of Figure 1 can be described. In the accepted normalization, quasistationary equations for amplitudes $\rho(\tau)$, $e'(\tau)$ and the envelope of the population difference $n(\tau)$ are written as follows:

$$\frac{d\rho'}{dt} - \frac{1+\kappa n}{\tau_2}\rho' + i(\Delta + \gamma n)\rho' = ne'_i,$$

$$\frac{dn}{dt} + \frac{n - n_0}{\tau_1} = -\frac{1}{2}\left[\rho'^*\left(e'_i - \frac{\kappa}{\tau_2}\rho'\right) + \rho'\left(e'_i - \frac{\kappa}{\tau_2}\rho'^*\right)\right].$$

It is further assumed that the probability amplitude of polarization can be represented as $\rho' = R + iS$ and $e'_i(t) = e_0$. Accordingly, the kinetic system for these variables is presented in this form [6]:

$$\frac{dR}{dt} = ne_0 - \frac{1+\kappa n}{\tau_2}R - (\Delta + \gamma n)S, \quad \frac{dR}{dt} = (\Delta + \gamma n)R - \frac{1+\kappa n}{\tau_2}S,$$
$$\frac{dn}{dt} = \frac{1-n}{\tau_1} - e_0 R + \frac{\kappa}{\tau_2}(R^2 + S^2)\ . \qquad (2)$$

The expressions for the equilibrium states R_S, S_S and n_S of system (2) are not difficult to determine from the singular limits of the corresponding equations (the expression for the dependence e_0^2 on n_S):

$$R_S = \frac{n_S^2}{Z\tau_2^2}e_0, \qquad S_S = \frac{\Delta + \gamma n_s}{Z\tau_2^2}n_s e_0,$$
$$e_0^2 = \frac{1-n_s}{\tau_1 n_s}\tau_2 Z, \quad Z = \frac{(1+\kappa n_s)^2}{\tau_2^2} + (\Delta + \gamma n_s)^2. \qquad (3)$$

From expressions (3) it follows that with certain combinations of coefficients (2) the stationary value of ns can be determined depending on the magnitude of the power e_0^2 ambiguously. It is known that the equilibrium states of models describing radiation when it is nonlinear resonantly interacting with a thin polarizable layer are characterized by a special property – the so-called bistability [7].

It is easy to verify that after separation of the real root, the characteristic equation for the exponent index λ, which determines the temporal dynamics of solutions for relatively small variations of the variables ΔR, ΔS and Δn in the neighborhood of (3) with the factor $\exp(\lambda\tau)$, is represented as:

$$\lambda^2 - \left[A - B + \frac{2}{3}\left(2Mn_S + \frac{1}{\tau_1}\right)\right]\lambda + \left[\frac{A-B}{2} + \frac{1}{3}\left(2Mn_S + \frac{1}{\tau_1}\right)\right]^2 + \frac{3}{4}(A+B)^2 = 0. \qquad (4)$$

Here

$$A = \left(C + \sqrt{C^2 + D^3}\right)^{\frac{1}{3}},\ B = \left(-C + \sqrt{C^2 + D^3}\right)^{\frac{1}{3}},\ C = \frac{1}{2}\left[\gamma(mr - pu) - M(\gamma n_S)^2 - \frac{QM}{3} - \frac{2M^3}{27}\right],$$

$$D = \left(M^2/3 - Q\right)/3, \quad M = \frac{\kappa n_S}{\tau_2} - \frac{1}{\tau_1}, \quad Q = (pr - mu)e_0^2.$$

The quantities $p = 1 - \frac{n_S}{\tau_2 Z}\left[\frac{\kappa}{\tau_2}\frac{1+\kappa n_S}{\tau_2} + \gamma\left(\Delta + \gamma n_S\right)\right]$, $m = \frac{n_S}{\tau_2^2 Z}\left(\gamma + \kappa\Delta\right)$, $u = \frac{\kappa n_S}{\tau_2^2 Z}\left(\Delta + \gamma n_S\right)$, and $u = \frac{\kappa n_S}{\tau_2^2 Z}\left(\Delta + \gamma n_S\right)$, included in parameters of equation (4) are also expressed in terms of the coefficients of equations (2).

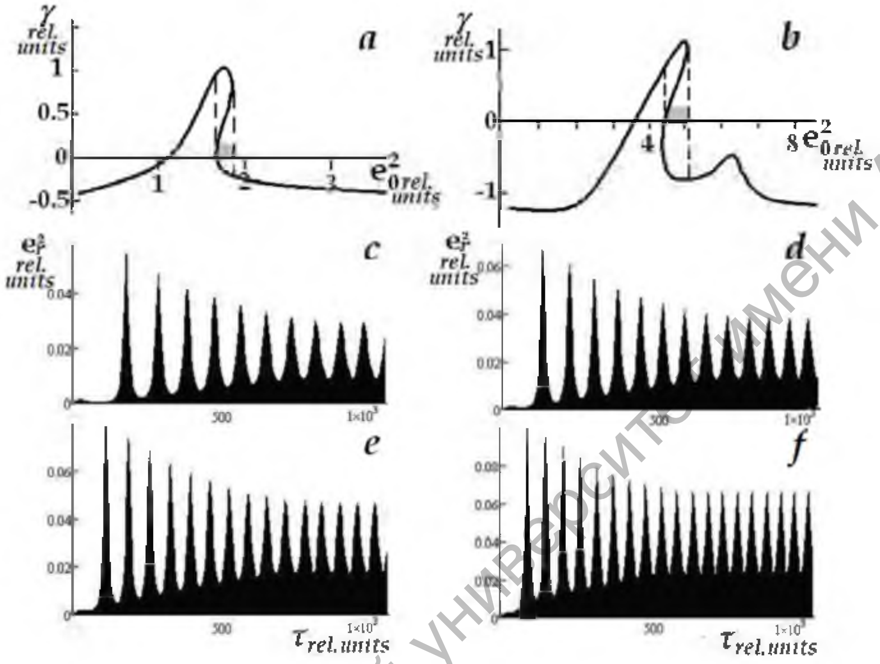

Fig. 2 – The dependence of the real part of the root of the characteristic equation (4) on the excitation parameter (*a*, *b*) and the temporal dependence of the normalized intensity of self-pulsations in the reflected radiation (*c* – *f*);
κ= 12.5 (*a, c*), *d* – κ= 15.0; κ= 16.0 (*b, ∂*); κ= 18.5 (*f*); e'_0=1.3 (*c, e*), 2.2 (*d, f*); γ= 0.08, τ_2=500, ω_0=1.45·10^14 *рад/с*

The complex roots of equation (4) can be expressed as follows:

$$\lambda_{1,2} = \frac{1}{2}\left[A - B + \frac{2}{3}\left(2\frac{\kappa n_S}{\tau_2} - \frac{1}{\tau_1}\right)\right] \pm i\frac{\sqrt{3}}{2}\left(A + B\right).$$

Thus, if the following relation

$$\gamma = A - B + \frac{2}{3}\left(2\frac{\kappa n_S}{\tau_2} - \frac{1}{\tau_1}\right) > 0, \qquad (5)$$

is true, the attractor of system (2) can be a limit cycle. The frequency Ω of cyclic motion of a point in phase space along a phase curve curled up to a limit cycle is calculated as $\Omega = \sqrt{3}(A+B)/2$.

Calculated estimates of the existence conditions of the self-oscillating regime of the nutational instability of variables based on (3), (4), (5) are more convenient to carry out parametrically, that is, setting the parameter linearly increasing within (0, 1) (Fig. 2, *a*, *b*). It should be noted that the dependence curves γ (e_0^2) are resonant in nature. In this case, the resonance is deformed in such a way that the dependence γ (e_0^2) in a certain region of the excitation parameter e_0 is ambiguous (this region is highlighted in the dependences shown in Fig. 2, *a*, *b*). This expresses bistability property mentioned above and such feature of equilibrium state characteristic, in general, characterizes the possibility of spontaneous instability and the transition of the nonlinear oscillatory system excited in such a bistability range to the self-oscillation regime.

Variants of the modeling of the dependence $e_r^2(\tau)$ basing on scheme (1) in Fig. 2, *c* – *f* were obtained for system parameters (1), approximately corresponding to condition (5) of the instability of the equilibrium state. It is clearly noticeable that the carrier oscillations with the optical frequency after the transitional evolution stage are modulated by the lower-frequency periodic envelope of nutational origin.

Conclusion

Obviously, nutation vibrations can cause a regular pattern of intensity in radiation reflected by a thin layer of a semiconductor structure with quantum-well effects modeled by a dense resonant medium. The occurrence of a series of self-sustaining pulsations is a consequence of the nonlinear phase shift of the absorption spectral line caused by the mutual influence of the near fields of dipoles under conditions of their relatively high concentration. The balance of oscillations of the resonance response generated by the optical nutation of dipole active centers and the effect of dipole-dipole interaction can lead to regular modulation of the initially continuous probing signal even under conditions of a certain influence of irreversible phase relaxation. This property of resonant reflection can be taken into account when developing lasers emitting series of short pulses with controlled parameters.

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Контакты: +375 222 60 41 37 (Тимощенко Елена Валерьевна)

Тимощенко Е. В., Юревич Ю.В. МОДЕЛИРОВАНИЕ НУТАЦИОННЫХ КОЛЕБАНИЙ В ИЗЛУЧЕНИИ, ОТРАЖЕННОМ ТОНКИМ РЕЗОНАНСНЫМ СЛОЕМ.

В рамках полуклассической осцилляторной модели с применением методов математической теории устойчивости динамических систем и численного анализа предсказана возможность модуляции излучения при резонансном отражении в режиме когерентного взаимодействия поля с веществом тонкого слоя.

Ключевые слова: тонкий планарный слой резонансных атомов, диполь-дипольное взаимодействие, самопульсации излучения.