DIOPHANTINE APPROXIMATIONS IN THE FIELD OF REAL AND COMPLEX NUMBERS AND HAUSDORFF DIMENSION

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Let $\mathcal{L}_1(\omega)$ be the set of real numbers for which inequalities

$$|\alpha - p/q| < q^{-\omega_1 - 1} \ or \ |\alpha q - p| < q^{-\omega_1}$$

have infinitely many solutions in integer p and natural q. Yarnik and Besikovitch found out that the Hausdorff dimension of $\mathcal{L}_1(\omega)$ equals $\frac{2}{\omega_1+1}$ when $\omega \geqslant 1$. This result was generalized for polynomials of arbitrary degree.

Let $\mathcal{L}_n(\omega)$ denote the set of $x \in \mathbb{R}$ such that the following inequality has infinitely many solutions in integer polynomials $P(x) \subset \mathbb{Z}[x]$:

$$|P_n(x)| = |a_n x^n + \cdots + a_1 x + a_0| < H^{-\omega_n}, \; H = \max_{0 \leqslant j \leqslant n} |a_j|$$

In [1] a lower estimate for $\dim \mathcal{L}_n(\omega_n)$ when $\omega > n$ is obtained, and V. Bernik obtained an upper estimate (see [2]). Based on their works we can conclude that $\dim \mathcal{L}_n(\omega_n) = \frac{n+1}{\omega+1}$ if $\omega_n > n$.

Earlier the generalization was obtained for the case of complex numbers.

The following generalization of the mentioned results for simultaneous approximations in $\mathbb{R} \times \mathbb{C}$ has been proved.

Let $S_n(\omega)$ denote the set $(x, z) \in \mathbb{R} \times \mathbb{C}$, such that the system of inequalities

$$\max(|P_n(x)|, |P_n(z)|) < H^{-\omega}$$

has infinitely many solutions in $P_n(t) \in \mathbb{Z}[t]$.

Theorem 1. *There exists a constant c that doesn't depend on n such that if $n \geqslant 3$ and $\omega > \frac{n-2}{3}$ then*

$$\dim S_n(\omega) < c\frac{n+1}{\omega+1}$$

References

[1] Baker A., Scmidt W. Proc. Lond. Math. Soc. 1970, 21. p.. 1–11.
[2] Bernik V. Acta Arithm. 1983, 42, p.219–253.