УДК 51-7

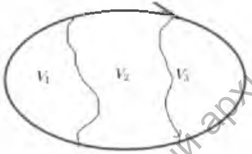
J. D. Ninova,
Sofia, Bulgaria

STRUCTURE OF THE SOLUTIONS OF SOME PROBLEMS

By using formal means of modelling conclusions, the article treats structures for the solutions of problems, in the conclusion of which notions are used, breaking trichotomy the volume of their common universal generic notion.

Keywords: relation between the volumes of three notions, modelling in the language of sets or in the language of propositions, derivation rules.

1. *Theoretical-didactic grounds*



The article discusses some didactic problems relating the notions, which are in the relation of opposition (as per the terminology and the classification described in [2]).

This relation can be presented by Venn's diagrams [2] and can be formally described by operations and re-

Электронный архив библиотеки МГУ имени А.А. Кулешова

$$x_0 \in U \Leftrightarrow x_0 \in V_1 \veebar x_0 \in V_2 \veebar x_0 \in V_3 \quad (1)$$

$$x_0 \in V_1 \Leftrightarrow x_0 \in U \wedge \overline{x_0 \in V_2 \cup V_3} \Leftrightarrow x_0 \in U \wedge \left(x_0 \notin V_2 \wedge x_0 \notin V_3\right) \quad (2)$$

$$\overline{x_0 \in V_1} \Leftrightarrow \overline{x_0 \in U \wedge \left(x_0 \notin V_2 \wedge x_0 \notin V_3\right)} \Leftrightarrow \overline{x_0 \in U} \vee \left(x_0 \in V_2 \vee x_0 \in V_3\right) \quad (3)$$

$$\overline{x_0 \in V_1} \Leftrightarrow x_0 \in V_2 \vee x_0 \in V_3 \quad (4)$$

lations with sets [3] in the following way:

$V_i \subset U,\ V_i \neq \varnothing,\ \ i = 1;2;3,\ V_i \cap V_j = \varnothing,\ i \neq j,\ \bigcup\limits_{1}^{3} V_i = U\ (*)$.

The following conditions can be added to the preceding conditions

$V_1 = U \setminus (V_2 \cup V_3);\ V_2 = U \setminus (V_1 \cup V_3);\ V_3 = U \setminus (V_1 \cup V_2)\ (**)$

$\overline{V_1} = V_2 \cup V_3;\ \overline{V_2} = V_1 \cup V_3;\ \overline{V_3} = V_1 \cup V_2\ (***)$,

which are not independent. The conditions (**) and (***) follow from the conditions (*).

Here is also a description of the situation using the propositional calculus.

The equivalence (1) describes the belonging of an object or of an n-let to the volume of the universal set. The equivalence (2) describes then belonging of an object or of an n-let to the volume of one of the notions, which are in this relation, i.e. it represents a model for generating examples from the volume of the respective notion. The equivalences (3) and (4) describe the non-belonging of an object or of an n-let to the volume of one of these notions, i.e. these are models for the generation of counter-examples.

The formalized models show that such examples for the learning of the respective notion can be consciously generated so that this didactical problem is turned into a standard didactical problem. In order not to generate counter-examples, which shall be beyond the volume of the generic notion, i.e. examples, which meet the requirement $\overline{x_0 \in U}$ of (3), then we accept in (4) that the object or the n-let belongs to the volume of the generic notion, but it belongs to the supplement of the respective set to the universal set.

The visual presentation of the relation of volumes' notions by Venn's diagrams and their formal description in the language of sets or in the language of propositions makes it possible to raise the research of didactical problems to a higher level, and to separate it from the specific knowledge. Thus the finding of solutions of didactical problems provides the opportunity for applying these solutions in each a specific case.

2. *Applications*

For the purpose of achieving the skills for using an indirect method of proving a statement, which can be re-formulated into an implicative form $(p \rightarrow q)$

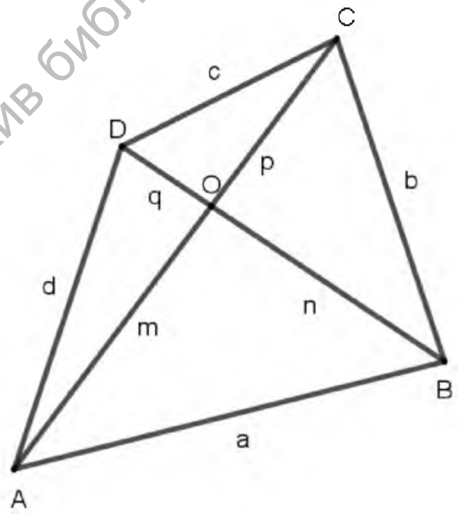
Электронный архив библиотеки МГУ имени А.А. Кулешова

, and in q a notion is used, which is in the relation of opposition, then one of the necessary skills is the skill of formation of a negation of reasoning [3] and equivalents of the same.

Structure of the solution of the problems following an indirect method, related with notions, which are in the discussed relation

We are solving a problem with structure $s \to t$ and $t \Leftrightarrow p\underline{\vee}q\underline{\vee}r$. We accept that the reasoning p has been executed. In such case, following the rule for the separation of disjunction, there follows the conclusion $q \vee r$ or $q\underline{\vee}r$, since both diagrams $\dfrac{p\underline{\vee}q\underline{\vee}r,\ p}{q\underline{\vee}r}$ (5) and $\dfrac{p\underline{\vee}q\underline{\vee}r,\ p}{q\vee r}$ (6) are derivation rules. The conclusions reveal the logical structure of the solution and reveal the number of the cases, which have to be considered for the purpose of solving the problem. In the following considerations the conclusions are made following diagram (6) due to the more often used laws for judgements with disjunctive structure. The assumption that the reasoning q is correct would lead to a contradiction. Consequently the reasoning $\bar{q}$ is correct. By analogy it can be established that also the reasoning $\bar{r}$ is correct. In such case, following one of the laws of De Morgan, it follows that $\bar{q} \wedge \bar{r} \Leftrightarrow \overline{q \vee r}$ (7) and as per diagram $\dfrac{p\underline{\vee}(q\underline{\vee}r),\ q\vee r}{p}$ (8) follows the correctness of reasoning p.

Example. If for the *sides* a, b, c *and d (taken in this order) of a quadrangle it is true that* $a^2 + c^2 = b^2 + d^2$ (s), then its *diagonals are perpendicular* (t).



Электронный архив библиотеки МГУ имени А.А. Кулешова

Comment. For angle φ between the diagonals of the quadrangle the following cases are possible: *p*: $\angle\varphi$ is right, *q*: $\angle\varphi$ is acute and *r*: $\angle\varphi$ is obtuse. The three notions acute angle, right angle and obtuse angle are in relation of opposition, if the universal set is the set of the elementary-geometric angles.

The arguments relating the solution of the problem specify the above described logical structure of the solution of problems with structure $s \to t$ and $t \Leftrightarrow p \veebar q \veebar r$. Let us accept that statement $\overline{p}$ is fulfilled, i.e. $\angle\varphi$ is not right. In such case as per diagram (6) it follows that we have to consider two cases. Let us assume that statement $q \vee r$ is correct, i.e. 1) *q*: $\angle\varphi$ is acute or 2) *r*: $\angle\varphi$ is obtuse.

In case $\angle\varphi$ is acute, i.e. we assume that reasoning *q* is correct. If $AC \cap BD = O, AO = m, BO = n, CO = p$ and $DO = q$, then from ΔAOB and ΔCOD it follows that $a^2 < m^2 + n^2$ and $c^2 < p^2 + q^2$. Through addition of the inequalities member by member we obtain that $a^2 + c^2 < m^2 + n^2 + p^2 + q^2$ (9). From ΔBOC and ΔAOD by analogue we obtain that $b^2 + d^2 > m^2 + n^2 + p^2 + q^2$ (10). From the derivations (9) and (10) and from the transitivity characteristic of the relation „is less than“, it follows that $a^2 + c^2 < b^2 + d^2$, which contradicts the fact that $a^2 + c^2 = b^2 + d^2$, i.e. reasoning $\overline{q}$ is correct. By analogue it is proven that also statement r is correct. Then, on the basis of law (7) and the derivation rule (8) follows the correctness of statement *p*, i.e. $\angle\varphi$ is right.

Of an analogical structure is part of the arguments, by which it is proven that the number $\sqrt{3}$ is irrational. Of an analogical structure is part of the arguments for the geometrical indirect proof of the theorem of Lehmus−Steiner in [1].

3. *Conclusion*

1. The use of formal models makes it possible to reveal the structure of the solution of the problem. Through the derivation rules (5), (6) and (8) strictly determined transfers are executed, considered from a terminological as well as from a logical point of view. The structure of the conclusions defines the number of the cases, which have to be discussed for the solution of the problem or for proving the statement, and the respective reasoning is are formulated in under the terms of the determined notions.

2. In view of the complex logical structure of such reasoning the teacher have to choose the form of activity – collective or independent, when choos-

Электронный архив библиотеки МГУ имени А.А. Кулешова

ing the indirect method for proving of statements of implicative structure, in the conclusion of which we operate with notions, which are in the discussed relation.

List of the literature used

1. Kang, K. H. Steiner−Lehmus Theorem / K. H. Kang, K. M. Koh // Mathematical Medley. − 1995. − № 1. − P. 37−39.
2. Nikitin, V. V. Definitions of Mathematical Notions in the Secondary School Course / V. V. Nikitin, K. A. Rupassov. – M. : Government of Education, 1963.− 150 p.
3. Ninova, J. D. Regarding the Learning of Some Mathematical Notions / J. D. Ninova // Mediterranean Journal for Research in Mathematics. − 2005. − № 1. − P. 103−117.

Электронный архив библиотеки МГУ имени А.А. Кулешова